One may classify fluids in two different ways:

- either according to their response to the externally applied pressure or
- according to the effects produced under the action of a shear stress.

**Classification of fluid behavior**

- *Newtonian*
- *Non-Newtonian*
Newtonian fluids

Consider a thin layer of a fluid contained between two parallel planes a distance dy apart

\[ \frac{F}{A} = \tau_{yx} = \mu \left( -\frac{dV_x}{dy} \right) = \mu \dot{\gamma}_{yx} \]

Newtonian fluids

- At any shear plane there are two equal and opposite shear stress
  - a positive one on the slower moving fluid and
  - a negative on the faster moving fluid layer

- The Negative sign on the right hand side of equation indicates that shear stress is a measure of the resistance to motion.
Newtonian fluids

For an incompressible fluid of density $\rho$

$$
\tau_{yx} = \left(-\frac{\mu}{\rho}\right) \frac{d}{dy} (\rho V_x)
$$

The quantity $\rho V_x$ is the linear momentum in the $x$ direction per unit volume (momentum concentration).

$\tau_{yx}$ represent the momentum flux in $y$ direction.

The negative sign indicates that the momentum transfer occurs in the direction of decreasing velocity.

Newtonian fluids

The constant of proportionality, $\mu$, is called Newtonian viscosity.

Independent of shear rate or shear stress.

Depends only on the material and its $T$ and $P$.

The plot of shear stress against shear rate for a Newtonian fluid, flow curve or rheogram is a straight line of slope, $\mu$, and passing through the origin.
Newtonian fluids

Gases, simple organic liquids, solution of low molecular weight inorganic salts, molten metal and salts are Newtonian fluids.

Typical shear stress–shear rate data for a cooking oil and a corn syrup
Newtonian fluids

For the more complex case of three-dimensional flow, it is necessary to set up the appropriate partial differential equations. For instance, the more general case of an incompressible Newtonian fluid may be expressed – for the x-plane (area oriented normal to the x-direction) – as follows (Bird et al., 1987, 2002):

\[
\tau_{xx} = -2\mu \frac{\partial V_x}{\partial x} + \frac{2}{3} \mu \left( \frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial z} \right)
\]

\[
\tau_{xy} = -\mu \left( \frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x} \right)
\]

\[
\tau_{xz} = -\mu \left( \frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x} \right)
\]

Stress components in three-dimensional flow

By considering the equilibrium of a fluid element, it can be shown that:

\[
\tau_{yx} = \tau_{xy} \quad \tau_{xz} = \tau_{zx} \quad \tau_{yz} = \tau_{zy}
\]
Stress components in three-dimensional flow

The normal stresses can be visualized as being made up of two components:

- isotropic pressure
- a contribution due to flow

\[
\begin{align*}
P_{xx} &= -p + \tau_{xx} \\
P_{yy} &= -p + \tau_{yy} \\
P_{zz} &= -p + \tau_{zz}
\end{align*}
\]

\[
p = -\frac{1}{3}(P_{xx} + P_{yy} + P_{zz})
\]

For an incompressible Newtonian fluid, the isotropic pressure is given by:

\[
p = -\frac{1}{3}(P_{xx} + P_{yy} + P_{zz})
\]
Non-Newtonian fluids

- Time independent fluids (purely viscous, inelastic, or generalized Newtonian fluids (GNF))
- Time dependent fluids
- Visco-elastic fluids

Time-independent fluid behaviour

In simple shear, the flow behaviour of this class of materials may be described by a constitutive relation of the form,

\[
\dot{\gamma}_{yx} = f(\tau_{yx}) \\
\tau_{yx} = f_1(\dot{\gamma}_{yx})
\]

This equation implies that the value of the shear rate at any point within the sheared fluid is determined only by the current value of shear stress at that point or vice versa.
Time-independent fluid behaviour

these fluids may be further subdivided into three types:

(a) Shear-thinning or pseudoplastic
(b) Viscoplastic
(c) Shear-thickening or dilatant.

Types of time-independent flow behaviour
Shear-thinning or pseudoplastic fluids

Shear stress (log scale)

Apparent viscosity (log scale)

Shear rate (s⁻¹)

Power-law model
Brookfield viscometer
Cone and plate viscometer
Capillary viscometer

μ₀
μ∞
Shear-thinning or pseudoplastic fluids

Mathematical models for shear-thinning fluid behaviour

More widely used viscosity models:

i. The power-law or Ostwald de Waele model

ii. The Carreau viscosity equation

iii. The Cross viscosity equation

iv. The Ellis fluid model
The power-law or Ostwald de Waele model

An expression of the following form is applicable:

$$\tau_{yx} = m(\dot{\gamma}_{yx})^n$$

Thus the apparent viscosity is given by:

$$\mu_{app} = \frac{\tau_{yx}}{\dot{\gamma}_{yx}} = m(\dot{\gamma}_{yx})^{n-1}$$

m: fluid consistency coefficient
n: flow behavior index

For n<1, the fluid exhibits shear-thinning properties
n=1, the fluid show Newtonian behavior
n>1, the fluid shows shear thickening behavior

<table>
<thead>
<tr>
<th>System</th>
<th>Temperature (K)</th>
<th>n (-)</th>
<th>m (Pa s^n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agro- and food-related products</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aerated poultry waste</td>
<td>283–296</td>
<td>1.81–0.161</td>
<td>1.12×10^{-3} (xy)^{0.75}</td>
</tr>
<tr>
<td>Ammonium alginate solution (3.37%)</td>
<td>297</td>
<td>0.5</td>
<td>13</td>
</tr>
<tr>
<td>Apple juice</td>
<td>–</td>
<td>0.13</td>
<td>200</td>
</tr>
<tr>
<td>Apple sauce</td>
<td>300</td>
<td>0.3–0.45</td>
<td>12–22</td>
</tr>
<tr>
<td>Apricot puree</td>
<td>300</td>
<td>0.3–0.4</td>
<td>5–20</td>
</tr>
<tr>
<td>Banana puree</td>
<td>293–315</td>
<td>0.33–0.5</td>
<td>4–10</td>
</tr>
<tr>
<td>Carrot puree</td>
<td>296</td>
<td>0.25</td>
<td>25</td>
</tr>
<tr>
<td>Chicken (minced)</td>
<td>296</td>
<td>0.10</td>
<td>900</td>
</tr>
<tr>
<td>Chocolate</td>
<td>300</td>
<td>0.5</td>
<td>0.7</td>
</tr>
<tr>
<td>Guava puree</td>
<td>296.5</td>
<td>0.5</td>
<td>49</td>
</tr>
<tr>
<td>Human blood</td>
<td>300</td>
<td>0.9</td>
<td>0.004</td>
</tr>
<tr>
<td>Mango pulp</td>
<td>300–340</td>
<td>0.3</td>
<td>3–10</td>
</tr>
<tr>
<td>Maraschino cream</td>
<td>–</td>
<td>0.4</td>
<td>560</td>
</tr>
<tr>
<td>Mayonnaise</td>
<td>746</td>
<td>0.6</td>
<td>5.4×10^{1}</td>
</tr>
<tr>
<td>Papaya puree</td>
<td>300</td>
<td>0.5</td>
<td>19</td>
</tr>
<tr>
<td>Peach puree</td>
<td>300</td>
<td>0.38</td>
<td>1–5</td>
</tr>
<tr>
<td>Peanut butter</td>
<td>–</td>
<td>0.07</td>
<td>500</td>
</tr>
<tr>
<td>Pear puree</td>
<td>300</td>
<td>0.4–0.5</td>
<td>1–5</td>
</tr>
<tr>
<td>Plum puree</td>
<td>287</td>
<td>0.35</td>
<td>30–80</td>
</tr>
<tr>
<td>Tomato concentrate</td>
<td>305</td>
<td>0.6</td>
<td>0.22</td>
</tr>
<tr>
<td>(5.3% solid)</td>
<td>295</td>
<td>0.24</td>
<td>33</td>
</tr>
<tr>
<td>Tomato ketchup</td>
<td>–</td>
<td>0.5</td>
<td>15</td>
</tr>
<tr>
<td>Whipped cream</td>
<td>297</td>
<td>0.5–0.6</td>
<td>15</td>
</tr>
<tr>
<td>Yoghurt</td>
<td>293</td>
<td>0.5–0.6</td>
<td>25</td>
</tr>
</tbody>
</table>
The Carreau viscosity equation

When there are significant deviations from the power-law model at very high and very low shear rates, it is necessary to use a model which takes account of the limiting values of viscosities $\mu_0$ and $\mu_\infty$

$$\frac{\mu_{\text{app}} - \mu_\infty}{\mu_0 - \mu_\infty} = \left\{1 + (\lambda \dot{\gamma}_{yx})^2\right\}^{(n-1)/2}$$

The Carreau viscosity equation

Another four parameter model (Cross 1965), which in simple shear, is written as:

$$\frac{\mu_{\text{app}} - \mu_\infty}{\mu_0 - \mu_\infty} = \frac{1}{1 + k(\dot{\gamma}_{yx})^n}$$

$n (<1)$ and $k$ are two fitting parameters whereas $\mu_0$ and $\mu_\infty$ are the limiting values of the apparent viscosity at low and high shear rates, respectively.
The Ellis fluid model

When the deviations from the power-law model are significant only at low shear rates, it is perhaps more appropriate to use the Ellis model.

$$\mu_{app} = \frac{\mu_0}{1 + \left(\frac{\tau_{yx}}{\tau_{1/2}}\right)^{\alpha-1}}$$

When the deviations from the power-law model are significant only at low shear rates, it is perhaps more appropriate to use the Ellis model.